Your Name

Good luck:) !!

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 484 of the text without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
Total	90	

1. (10 total points) Evaluate the following indefinite integrals.

   (a) (5 points) $\int \sqrt{x}e^{\sqrt{x}}\,dx$

   (b) (5 points) $\int \frac{x^2-2x-1}{(x-1)^2(x^2+1)}\,dx$

2. (10 total points) Evaluate the following definite integrals.

   (a) (5 points) $\int_0^1 \ln(1+t^2)\,dt$ *Hint: Use integration by parts*

   (b) (5 points) $\int_{-3}^{-1} \frac{1}{(t^2+4t+5)^{3/2}}\,dx$

3. (10 points) Determine whether the following improper integral is convergent or divergent. If it is convergent, evaluate it.

$$\int_0^{\infty} \frac{e^x}{e^{2x}+3}\,dx.$$

4. (10 points) A particle is moving along a straight line. For time $t \geq 0$, the velocity of the particle is given by

$$v(t) = t^2 - 2t + 1.$$

Find the *total distance* traveled by the particle from time $t = 0$ to time $t = 5$. Set up the integral computing the *displacement* of the particle during $t = 0$ to $t = 5$.

5. (10 total points) The triangle whose vertices have (x,y) coordinates (0,2), (1,0) and (2,2) is rotated around the vertical line $x = -1$ to form a solid of revolution.

   (a) (5 points) Using *SLICING*, set up a definite integral (or the sum of two definite integrals if necessary) for the volume of this solid of revolution. DO NOT EVALUATE THE INTEGRAL(S).

   (b) (5 points) Using *SHELLS*, set up a definite integral (or the sum of two definite integrals if necessary) for the volume of this solid of revolution. DO NOT EVALUATE THE INTEGRAL(S).

6. (10 total points) The concentration of a chemical solution satisfies the differential equation

$$\frac{dC}{dt} = r - kC,$$

where $C = C(t)$ denotes the concentration at time t, r and k are positive constants.

(a) (7 points) Solve the differential equation to find all the solutions.

(b) (3 points) The concentration at time $t = 0$ is C_0. Determine the concentration at any time t by solving the initial value problem.

7. (10 total points) Find the values of x for which the following series' converge. Find the sum of the series for those values of x if the series is geometric.

   (a) (5 points)

   $$\sum_{n=0}^{\infty}(-1)^n\frac{(3x-4)^n}{3^{2n}}$$

   (b) (5 points)

   $$\sum_{n=0}^{\infty}\frac{n!x^n}{1\cdot 3\cdot 5\cdots(2n-1)}$$

8. Find the Maclaurin series and their radii of convergence:

By definition, the Maclaurin series is computed from $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$. Therefore, to compute the Maclaurin series by definition, we need to compute all the derivatives of the function evaluated at zero. However, by the computation of the coefficients, we obtain the uniqueness of the series, so as long as the power series is of the above form, it is automatically the Maclaurin series. The following solutions are computed via different methods from the definition.

(a) (5 points) Find the Maclaurin series for $f(x) = x\cos(\frac{1}{2}x^2)$, given $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$

Proof. Replace x by $\frac{1}{2}x^2$ in $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$ to get the Maclaurin series of $\cos(\frac{1}{2}x^2)$, then multiply both sides by x to get the Maclaurin series of

$$f(x) = x\cos(\frac{1}{2}x^2) = x\sum_{n=0}^{\infty}(-1)^n \frac{(\frac{1}{2}x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+1}}{2^{2n}(2n)!}$$

□

(b) (5 points) Find the Maclaurin series for $f(x) = \ln(1+x^2)$, given $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

Proof. Replace x by $-x$ in the Maclaurin series of $\frac{1}{1-x}$ to get the Maclaurin series of $\frac{1}{1+x}$. Integrate the series and obtain

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}(-1)^n$$

Finally, replace x by x^2 to get the Maclaurin series of

$$f(x) = \sum_{n=0}^{\infty} \frac{x^{2n+2}}{n+1}(-1)^n$$

□

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dx} = 2x e^{-tan(y)} \cos^2(y), \quad y(0) = \frac{\pi}{4}.$$

Give your answer in the form $y = f(x)$.